

Graph Theory and Combinatorics

Semestral Examination

April 29, 2025

Instructions: All questions carry ten marks.

1. Let n be a natural number. Define affine and projective plane of order n . Prove that a projective plane of order n exists if and only if an affine plane of order n exist.
2. Let G be a simple graph on n vertices with $n > 3$. Assume that G has no vertex of degree $n - 1$ and any two distinct vertices of G have a unique common neighbour. Prove that G is a regular graph.
3. Prove that every planar graph is 5 vertex colourable.
4. Let G be a connected k -regular graph. Prove that k is the maximum eigen value of its adjacency matrix and that it occurs with multiplicity one.
5. Let (X_1, Y_1) and (X_2, Y_2) be minimum cuts in a transportation network. Prove that $(X_1 \cup X_2, Y_1 \cap Y_2)$ is also a minimum cut.
6. Let C be a perfect binary e -error correcting code of length n . Assume that $0 \in C$. Let \mathcal{B} denote the collection of subsets of $\mathcal{P} = \{1, \dots, n\}$ formed by supports of elements of C that have weight $2e + 1$. Prove that $(\mathcal{P}, \mathcal{B})$ form a $(e + 1) - (n, 2e + 1, 1)$ design.